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A RATIONAL CURVE RELATING LENGTH OF REST PERIOD  
AND LENGTH OF SUBSEQUENT WORK PERIOD  
FOR AN ERGOGRAPHIC EXPERIMENT

*A Technical Report*

*prepared by*

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ABSTRACT

A rational function is developed relating the length of a rest period and length of subsequent work period in an ergographic situation. Simple energetic postulates are used for a critical organ or neuromuscular structure whose failure to perform adequately results in a stoppage of the work period. Experimental results for two subjects using a finger ergograph indicate that the function yields the general trend of the data but that there seems to be some systematic deviations of the data from the present rational function. One parameter determined from the data represents rate of recovery from moderate fatigue. It is hoped that this development will aid in studies of motor functions as related to such other variables as age, motivation, and effects of drugs.

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AND LENGTH OF SUBSEQUENT WORK PERIOD  
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The idea for the present rational development occurred during a perusal of general literature on work decrement. A number of psychologists have used the ergograph in a variety of studies ranging from those concerned with personality characteristics to those dealing with work in industry. While considerable progress has been made by physiologists on characteristics of active muscles and nerves, there seems to have been only moderate success in application of these physiological developments to the problems encountered by psychologists in dealing with behavior of integrated, intact individuals. Indeed, there are a number of instances where psychologists claim that behavior such as exhibited with the ergograph cannot be accounted for on purely physiological and energistic grounds. The difficulty may be in finding how the various physiological details can be incorporated into descriptions of behavior of the complete individual. A second possibility is that psychologists have not considered sufficiently simple and limited behavioral situations to observe the physiological and energistic determiners of behavior. In the present case a few simple energy relations are postulated which only approximate the relations that might be determined on physiological grounds. These simple relations, however, permit development of a functional relation observable in the performance of an individual in a limited ergographic experiment. Psychologists may find the present development of use in studying more complex situations.

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After an individual has performed a constant, repetitive, motor task to such an extent that he no longer can continue, a rest period will result in the individual's being able to perform the task again for some work period before again being unable to continue. A graph relating length of rest period and length of subsequent work period will be of the form of those shown in Figure 1. A similar result was obtained empirically by Manzer (2). Short rest periods will be followed by short work periods, longer rest periods will be followed by longer work periods. As the rest periods are lengthened, the subsequent work periods should also lengthen, but to a progressively lesser extent until some maximum length of work period is approached.

In developing a rational function, several assumptions are made concerning energy relations within the organ or neuromuscular structure whose failure to function adequately is responsible for the work stoppage. The organ might be the working muscle, or it might be one of the nervous elements responsible for excitation of the muscle. We will consider at this time only the critical organ or structure whose failure to function adequately results in failure of the individual to perform the task. It is assumed that fatigue of other organs or structures will have little effect on length of the work period so long as these organs do function. This is probably an oversimplification of the situation. Interaction between organs or structures probably does occur such that fatigue in one results in greater expenditure of energy by others in order for the individual to continue the task. This interaction is being ignored in the present development.

Consider an organ that is using energy at some constant rate during a working period. The supply of energy immediately available to the organ to be used in performing the task is being depleted. If this energy is being replenished at a slower rate than that at which it is being used, the supply of energy will be reduced. When the energy level falls to some critical point, the individual will be unable to continue the task and the work period will end.

During a rest period the energy supply of the organ will be replenished to an extent dependent on the length of the rest period and the rate at which energy is being made available to the organ by the rest of the individual's body. (For the present development, the nature of the physiological mechanism involved is not of immediate relevance.) At the end of such a rest period, the immediately available energy supply of the organ will again support performance of the task during a subsequent work period.

Consider the following postulates and definitions. Let:

$$E_t = \text{energy immediately available to organ at time } t; \quad (1)$$

$$E_m = \text{energy immediately available to organ when it is in a completely rested state; } \quad (2)$$

$$a = \text{rate of expenditure of energy during work period.} \quad (3)$$

This is postulated to be a constant;

$$C(E_m - E_t) = \text{postulated rate at which body replaces energy to the organ; } \quad (4)$$

$$W = \text{length of work period; and } \quad (5)$$

$$R = \text{length of rest period. } \quad (6)$$

It is to be noted that the postulate in equation 3 forms a limit on the type of situation to which the present development is appropriate.

The task should not be one for which the individual may work faster when more rested and then slow down when he becomes fatigued, nor should the task vary with the fatigue of the individual. The common type of ergographic series, where there may be long initial strokes followed by short strokes as the individual tires, is inappropriate for the present development. In an ergograph situation the strokes should be of constant length and made at constant timing. Inability to make a stroke of standard length is to be interpreted as failure to perform the task. Thus, the individual is not driven to such fatigue that he cannot make a stroke of any length; he just cannot make one of standard length. Even in this case, this assumption of a constant rate of expenditure of energy is probably an approximation.

The postulate in equation 4 involves the simple concept that energy replacement occurs at a rate proportional to the extent of deficiency below a maximum amount of energy available. This maximum amount of energy available is that which would be present in a completely rested organ.  $C$  is the constant of proportionality.  $(E_m - E_t)$  is the extent of energy deficiency. This postulate is probably a gross approximation to a true function which could be determined from physiological considerations but it should be usable for cruder developments and when a limited task is involved, such as the flexion of a finger. This postulate would probably be inappropriate for more extensive tasks involving a large proportion of the body.

Consider the curve of energy available versus time in Figure 2. At time  $t_0$  the energy level is  $E_0$ . As an initial work period progresses the energy level decreases along the curve until it reaches the level  $E_1$ . This energy level  $E_1$  is the critical value between continuation in performance of the task and discontinuation of performance. Whenever the energy level is below  $E_1$  there is insufficient energy available for the organ to continue

its part in the performance of the task. Whenever the energy level is above  $E_1$  there is sufficient energy for the organ to continue its activity. The time interval for this initial work period is  $W_0$ .

A rest period of duration  $R_1$  is now imposed and the energy level builds up to  $E_2$ . During the following work period the energy level reduces to  $E_3$ , a critical level between continued and non-continued performance of the task. Since the task has not been altered, we might postulate that:

$$E_3 = E_1 \quad (7)$$

The duration of this work period is  $W_1$ .

Consider that a second rest period of duration  $R_2$  is imposed which is followed by a work period of duration  $W_2$ . The terminal energy level  $E_5$  is again the critical level between continued and non-continued performance of the task; and, therefore, is equal to  $E_3$  and  $E_1$ . Let:

$$R_2 = R_1 \quad (8)$$

These two rest periods started with the identical energy levels  $E_3$  and  $E_1$  as postulated in equation 7; thus, if the energy restoration conditions are identical as postulated in equation 4, the energy levels at the end of these rest periods should be identical; that is:

$$E_2 = E_4 \quad (9)$$

It might be expected, then, that the two subsequent work periods would be identical also:

$$W_2 = W_1 \quad (10)$$

This logic would lead to an expectation that a long sequence of rest-work periods with equal rest periods would have equal work periods. Yochelson (3)

has reported data indicating such constancy in work periods following definite rest periods in long sequences of rest and work periods. Data gathered in the experimental try-out of this development also tended to support this contention. A finger ergograph working against spring tension with a fixed excursion to a block was used. The rate of contractions was set at one contraction per second. Preliminary trials revealed considerable initial practice effect, session to session, for a subject. During practice sessions some long sequences of rest and work periods with equal rest periods were tried out. The following results for the sixth practice session for one subject using a series of 60 second rest periods are typical. The lengths of the work periods, in seconds, were 54, 36, 30, 30, 31, 30, 28, 28, 28, 28, 33, 31, 30, 34, 28, 28, 31, 39, 28, 28. The first one of 54 seconds in this series should not be counted. It corresponds to the initial work period  $W_0$  before any of the fixed rest periods and might be expected to be long. The remaining work periods seem to vary within a fairly constant band with no apparent progressive decrement. Presumably this will hold only for a finite time and the experiment should not involve excessive sessions.

During the rest period  $R_1$  the rate of change of energy with time can be obtained from assumption (4). Only energy replacement is considered to be active during the rest period.

$$\frac{dE_t}{dt} = C(E_m - E_t) \quad (11)$$

Integration yields:

$$E_m - E_t = e^{(-Ct + f)} \quad , \quad (12)$$

where  $f$  is a constant of integration. When the terminal times  $t_1$  and  $t_2$  and the corresponding energy levels  $E_1$  and  $E_2$  are substituted into equation 12, the following ratio may be obtained:

$$\frac{E_m - E_1}{E_m - E_2} = \frac{e^{(-Ct_1 + f)}}{e^{(-Ct_2 + f)}} \quad (13)$$

$$= e^{(-Ct_1 + f + Ct_2 - f)} \quad (14)$$

$$= e^{C(t_2 - t_1)} \quad (15)$$

It is to be noted that the length of the rest period is

$$R = t_2 - t_1 \quad , \quad (16)$$

since  $t_2$  and  $t_1$  are the end and beginning times. The subscript to  $R$  is being dropped for convenience. Equation 15 can then be written:

$$\frac{E_m - E_1}{E_m - E_2} = e^{CR} ; \quad (17)$$

or, solving for  $E_2$ :

$$E_2 = E_m - (E_m - E_1)e^{-CR} \quad . \quad (18)$$

Consider the subsequent work period. Energy is being used at a constant rate as per assumption 3 as well as being replenished as per assumption 4. Thus:

$$\frac{dE_t}{dt} = -a + C(E_m - E_t) \quad . \quad (19)$$

Integration yields:

$$E_m - E_t - \frac{1}{C} a = e^{(-Ct + g)}, \quad (20)$$

where  $g$  is a constant of integration. Substitution of limiting times  $t_2$  and  $t_3$  and energy levels  $E_2$  and  $E_3$  and writing a ratio yields:

$$\frac{E_m - E_2 - \frac{1}{C} a}{E_m - E_3 - \frac{1}{C} a} = \frac{e^{(-Ct_2 + g)}}{e^{(-Ct_3 + g)}} \quad (21)$$

$$= e^{C(t_3 - t_2)} \quad (22)$$

$$= e^{CW}, \quad (23)$$

where

$$W = t_3 - t_2 \quad (24)$$

Substitution of  $E_1$  for  $E_3$  as per equation 7 and solution for  $E_2$  yields:

$$E_2 = E_m - \frac{1}{C} a - (E_m - E_1 - \frac{1}{C} a) e^{CW} \quad (25)$$

In relating the work period and rest period, the two expressions for  $E_2$  in equations 18 and 25 are equated, yielding:

$$E_m - (E_m - E_1) e^{-CR} = E_m - \frac{1}{C} a - (E_m - E_1 - \frac{1}{C} a) e^{CW} \quad (26)$$

Subtracting  $E_1$  from both sides of the equation,

$$E_m - E_1 - (E_m - E_1) e^{-CR} = E_m - E_1 - \frac{1}{C} a - (E_m - E_1 - \frac{1}{C} a) e^{CW} \quad (27)$$

Or:

$$(E_m - E_1) (1 - e^{-CR}) = (E_m - E_1 - \frac{1}{C}a) (1 - e^{CW}) \quad (28)$$

$$\frac{(E_m - E_1)}{(E_m - E_1 - \frac{1}{C}a)} (1 - e^{-CR}) = (1 - e^{CW}) \quad (29)$$

Define:

$$B = \frac{(E_m - E_1)}{(E_m - E_1 - \frac{1}{C}a)} \quad (30)$$

Then:

$$B(1 - e^{-CR}) = (1 - e^{CW}) \quad (31)$$

Or, solving for  $e^{CW}$ :

$$e^{CW} = 1 - B + Be^{-CR} \quad (32)$$

Thus  $e^{CW}$  is linearly related to  $e^{-CR}$  with a slope of B and intercept of  $1 - B$ .

It is interesting to note that when the numerator and denominator of the right side of equation 30 are multiplied by C

$$B = \frac{C(E_m - E_1)}{-a + C(E_m - E_1)} \quad (33)$$

Thus, from equations 11 and 19:

$$B = \frac{\frac{dE}{dt} \quad (\text{for the rest period})}{\frac{dE}{dt} \quad (\text{for the work period})} \quad (34)$$

Another point of interest is that the relation given in equation 31 (or 32) does not involve directly the amount of energy expended. Only measures of duration of rest and work periods need be determined. It is not necessary to observe the energy expended as is frequently attempted in ergographic experiments by computing the work performed by the muscle. (In case the muscle is not the critical organ responsible for the work stoppage, the work performed by the muscle would not be equal to the energy expenditure to be considered in case it were necessary to determine the constant a.) This fortunate feature is due to the restriction to a situation for which there is a constant rate of energy expenditure for the critical organ.

Three experimental sets of data were obtained. All used the finger ergograph previously described which involved a spring load rather than weights. The excursion of the finger tip was limited by a block. The rate of finger contractions was set at one per second in all three cases. On each contraction, the limit block was to be touched. Failure to make a complete stroke ended each work period. In each experiment one subject was used for a number of sessions. Each session was composed of a "warm-up" period involving three work periods separated by 60 second rest periods. The first experimental rest period followed immediately the last warm-up work period. In the experimental session proper, a sequence of rest, work, rest, work, etc. periods were used. Instead of having a sequence of equal rest periods and thus determining one point on the rest-work curve at each session, each of the selected rest periods was used once at each session and the subsequent length of work period was determined. The order of rest periods was varied between sessions. Mean length of work periods following each length of rest period was determined for each subject.

Data for the initial subject are not presented here because he showed considerable practice effect from session to session. The subject performed about twice as much work on the fourth session as on the first session. The other two subjects received more extensive practice sessions and showed less increase in work performed during the experimental sessions. Results for the preliminary subject were analyzed and the curve of equation 32 fits this data about as well as it does the data for the two subjects reported here.

Mean lengths of work periods following the chosen rest periods are given in Table 1.

The values of B and C for each subject were determined graphically. In cases where more precise determinations of these constants are desired, some more precise statistical method of curve fitting might be used. In the present case we were interested in obtaining only the proper order of magnitudes of B and C and felt that there was an advantage in the graphical method in surveying the properties of the function and the data. A series of trial values of C were assumed. For each value of C, values of  $e^{CW}$  and  $e^{-CR}$  were obtained.

Figure 3 shows graphs for subject 1 for three values of C. Each point is determined by one rest period and the subsequent work period. From equation 32 it is expected that the points between  $e^{CW}$  and  $e^{-CR}$  would be linearly related for the proper value of C. Analysis of equation 32 also indicates that this line should pass through the point 1, 1. All three lines drawn in Figure 3 pass through this point. It is to be noted in Figure 3 that a low value of C yields a negative curvature and a high value of C yields a positive curvature. A C of .010 seemed

to yield the best approximation to a straight line. A best fitting line was drawn by eye with a slope of -.76, thus determining B. The line drawn on Figure 1 for subject 1 is the line for equation 32 with the values of B and C determined above.

Figure 4 shows graphs of  $e^{CW}$  and  $e^{-CR}$  for subject 2 and three trial values for C. While deviations from a straight line do not seem extreme, it is of interest that the points cannot be brought into more or less random fluctuations around a straight line by any choice of C. There seems to be a systematic wave about the straight line in each graph. This type of curve may result from the inadequacies of our approximations. A slight suggestion of this effect may be detected also for subject 1. in Figure 3. These results seem to be related to the results reported by Féré (1) and by Manzer (2) where performances after moderate rest periods were superior to performances after longer rest periods. While the consistency and seriousness of this lack of fit of the present function is a matter for further study, it is the author's judgment from the present data that equation 32 yields the general sweep of the observations. Some different set of postulates might yield a better fit to the data, or the systematic deviations may be considered as perturbations to be accounted for by further complexities in the mathematical model.

Returning to the fitting of equation 32 to the results for subject 2, values of .008 for C and -.95 for B seemed to give the best fit to the data. The corresponding curve is drawn in Figure 1.

An asymptote is indicated for each curve in Figure 1. This asymptote may be determined by setting R equal to infinity in equation 32,  $e^{-CR}$  is then zero and  $e^{CW}$  equals  $(1 - B)$ . In the experiment, the first work

period exceeded this asymptote by some 10 to 20 per cent. This is a second indication of an inadequacy of our formulation which might be corrected by a more complex set of postulates. Another possibility is to interpret the present function to apply to the body state following the warm-up period in the experimental sessions.

Future work with the rational rest-work function can take any of several lines aside from development of a more adequate (and probably more complex) function. Individual differences in values of C and B for a fixed experiment might be correlated with other variables such as age. Effects of such conditions as ventilation, use of drugs, motivation, and response modality on C and B could be investigated. The experiment could be expanded to include systematic variation in load on the ergograph and timing of flexions, thus investigating other characteristics of the present function when  $a$  (rate of energy expenditure) and  $E_1$  (critical energy level) are varied. It would be hoped that use of the rational function for the rest-work curve would help in obtaining greater precision in results for these various types of investigations.

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TABLE 1

EXPERIMENTAL RESULTS  
 (All times are in seconds.)

Subject 1		Subject 2	
<u>Length of Rest</u>	<u>Length of Work*</u>	<u>Length of Rest</u>	<u>Length of Work**</u>
5	5.8	5	10.0
10	9.9	10	14.7
20	14.9	20	21.5
40	23.5	40	31.8
60	27.8	60	39.7
90	37.5	80	45.2
120	41.5	100	51.0
160	48.1	120	54.0
200	50.0	180	67.0
240	52.9	240	77.8

\*Mean length of work periods over 8 sessions.

\*\*Mean length of work periods over 6 sessions.

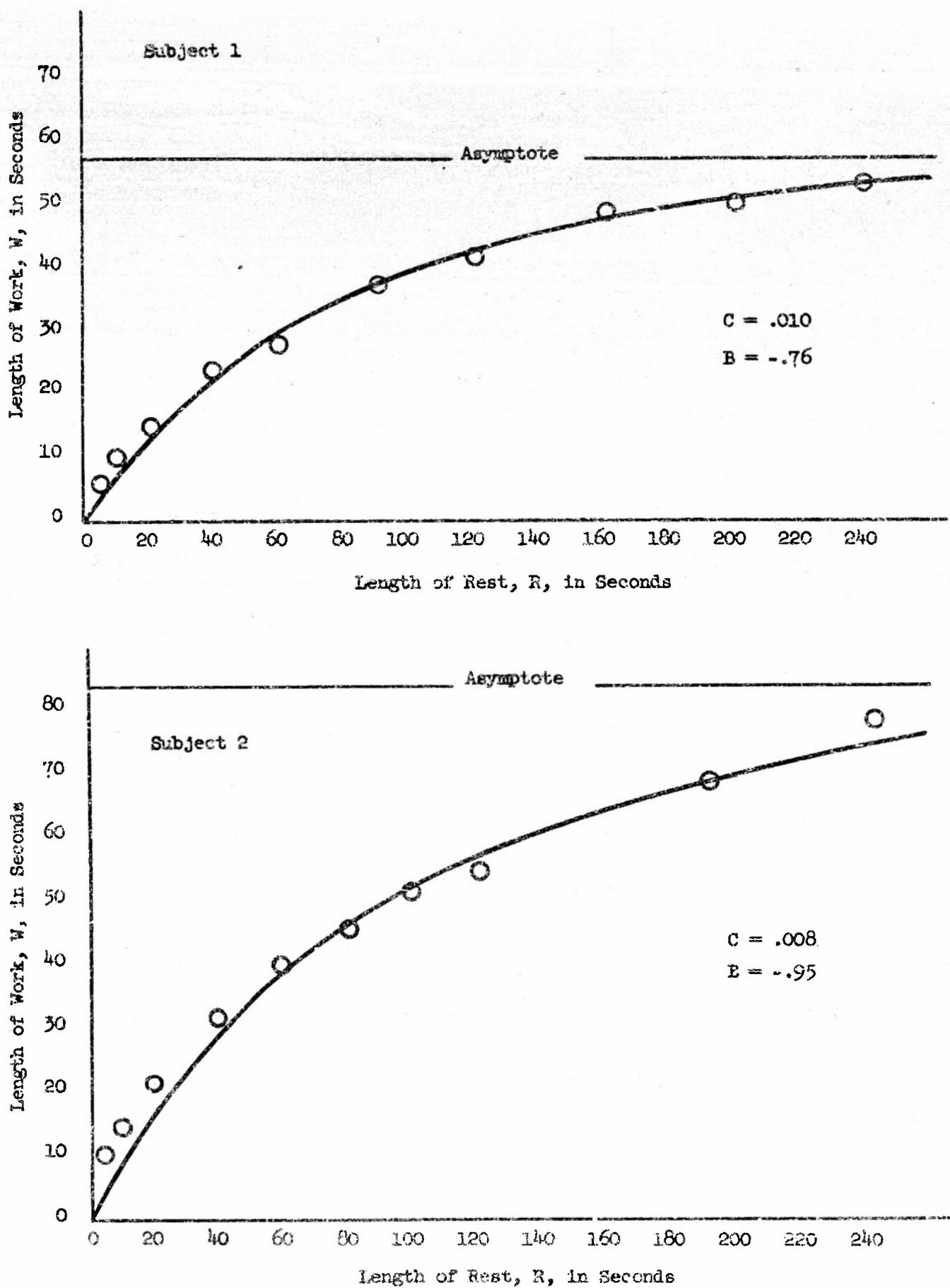


Figure 1. Rest-Work Curves for Two Subjects

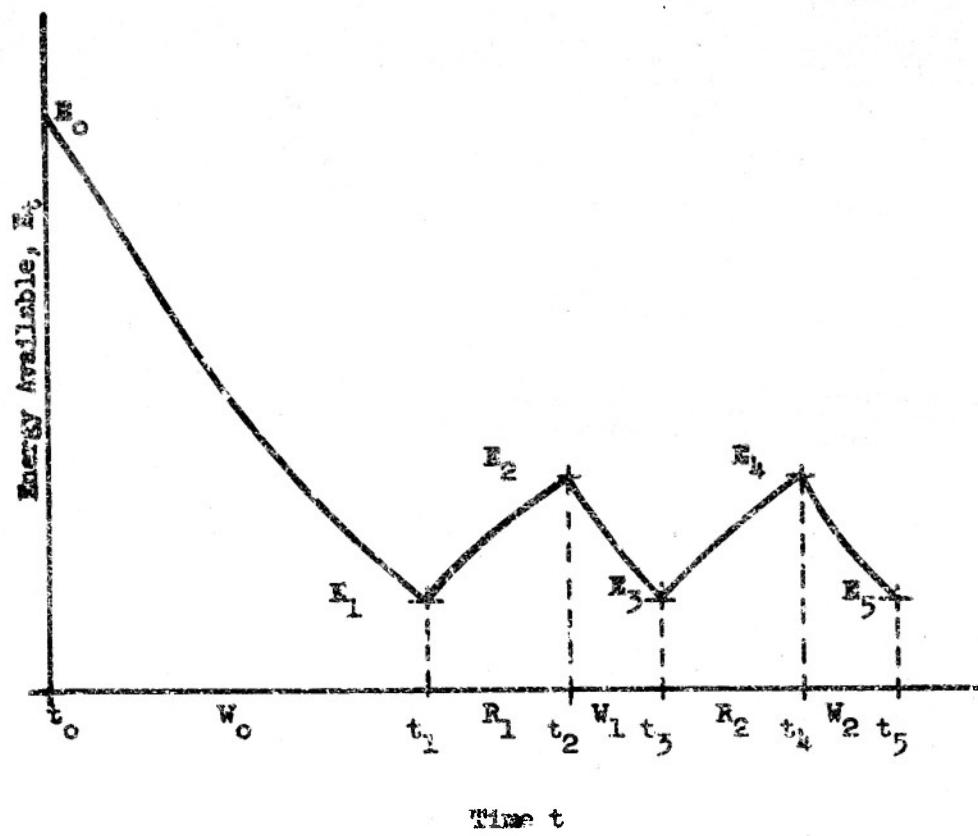


Figure 2

Relation Between Energy Available and Time  
During a Series of Work and Rest Periods.

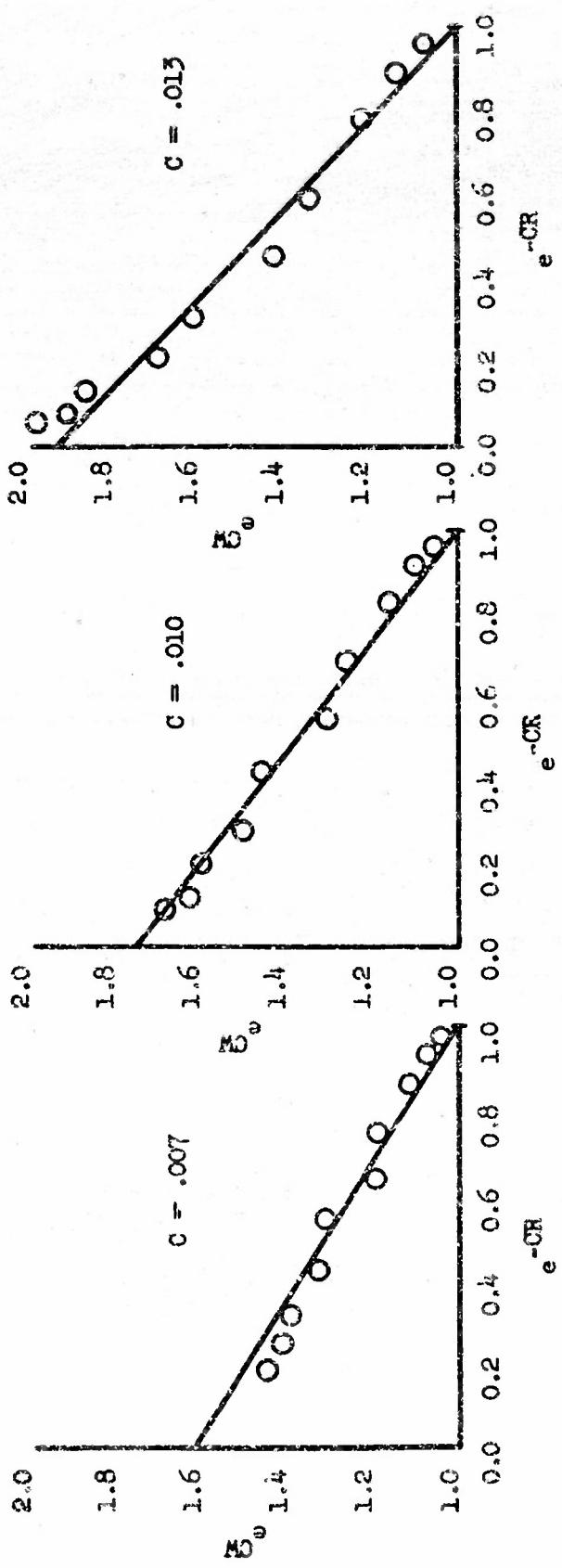


Figure 3. Rectification of Rest-Work Curve for Subject 1.

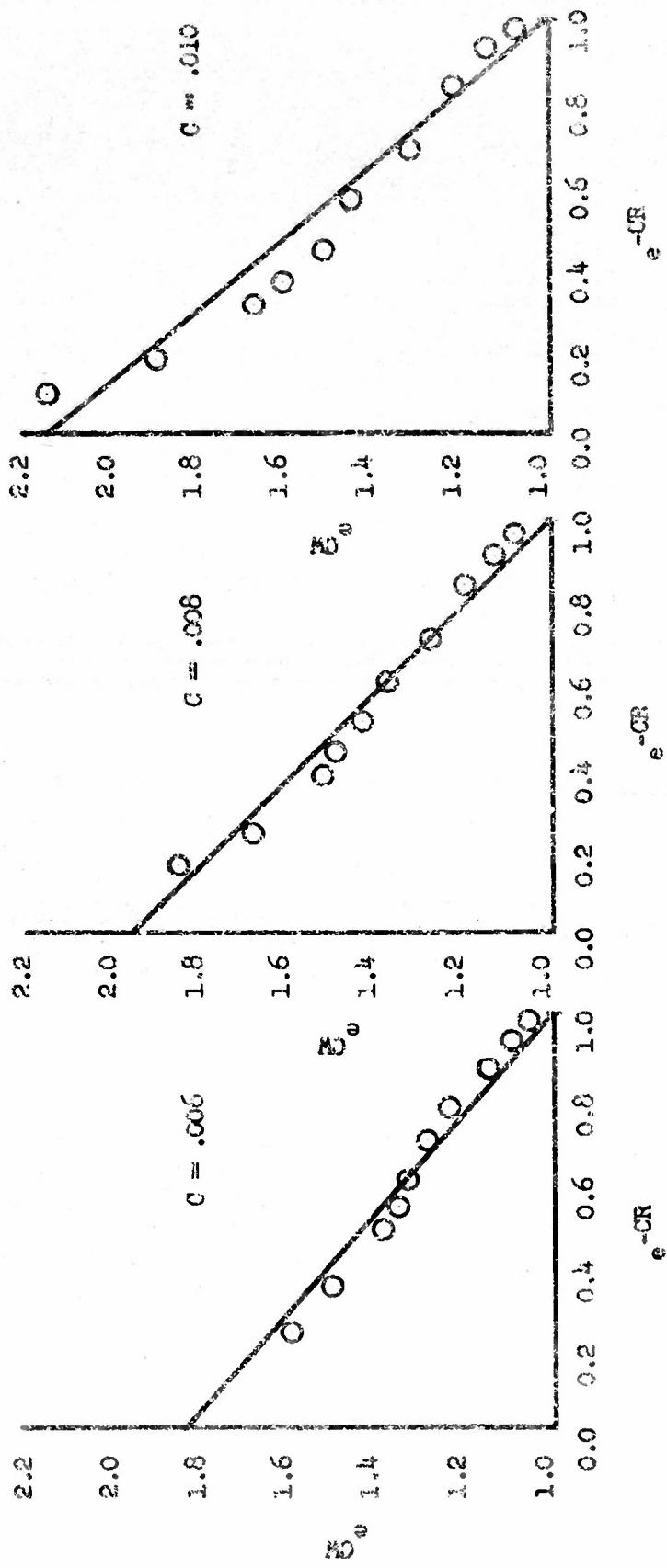


Figure 4. Rectification of Rest-Work Curve for Subject 2.

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